

1

$$\cos(\alpha + \beta) = \cos\alpha - 1 \cdots \cdots \textcircled{1}$$

$$\cos(\alpha + \beta) = \cos\beta - 1 \cdots \cdots \textcircled{2}$$

①, ②より

$$\cos\alpha - 1 = \cos\beta - 1$$

$$\cos\alpha = \cos\beta$$

$$0 \leq \alpha, \beta \leq \pi \text{ より } \alpha = \beta$$

① (又は②) は $\cos 2\alpha = \cos\alpha - 1$

$$\Leftrightarrow 2\cos^2\alpha - 1 = \cos\alpha - 1$$

$$2\cos^2\alpha - \cos\alpha = 0$$

$$(2\cos\alpha - 1)\cos\alpha = 0$$

$$\cos\alpha = \frac{1}{2}, 1$$

$$\alpha = \frac{\pi}{3}, \frac{\pi}{2}$$

$$(\alpha, \beta) = \left(\frac{\pi}{3}, \frac{\pi}{3} \right), \left(\frac{\pi}{2}, \frac{\pi}{2} \right)$$

(1)

2

(i) $y = x^2 - 2x - 3 = (x-1)^2 - 4$ 頂点(1, -4)

$y = x^2$ の焦点は $\left(0, \frac{1}{4} \right)$

なので,

$y = x^2 - 2x - 3$ の焦点は

$$\left(0+1, \frac{1}{4}-4 \right) = \left(1, -\frac{15}{4} \right)$$

(2)

⊛ 焦点が $(0, p)$
 準線が $y = -p$
 $\Rightarrow 4py = x^2$

<p>(ii) $y = x^2 - 2x - 3$</p> <p style="text-align: center;">↓ y 軸対称</p> <p>$y = x^2 + 2x - 3$</p> <p style="text-align: center;">↓ $y = x$ に関して対称</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $x = y^2 + 2y - 3$ </div> <p style="text-align: center;">(3)</p>	<p>$\left(1, -\frac{15}{4}\right)$</p> <p style="text-align: center;">↓ y 軸対称</p> <p>$\left(-1, -\frac{15}{4}\right)$</p> <p style="text-align: center;">↓ $y = x$ に関して対称</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $\left(-\frac{15}{4}, -1\right)$ </div> <p style="text-align: center;">(4)</p>
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3

(i) $W(x) = \int_0^x e^{-a(x-s)} f(s) ds = \int_0^x e^{-ax} \cdot e^{as} f(s) ds$

$$= e^{-ax} \int_0^x e^{as} f(s) ds$$

$$W'(x) = -ae^{-ax} \int_0^x e^{as} f(s) ds + e^{-ax} \times e^{ax} f(x)$$

$$= -a \int_0^x e^{-a(x-s)} f(s) ds + f(x)$$

||

$W(x)$ より

$$W'(x) = -aW(x) + f(x)$$

$$\Leftrightarrow \boxed{W'(x) + aW(x)} = f(x) \cdots \cdots \textcircled{1}$$

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(ii) $W'(x) + W(x) = x^2$

①と比較して $a=1$, $f(x) = x^2$ とする

$$W = \int_0^x e^{-(x-s)} f(s) ds$$

$$= e^{-x} \int_0^x e^s s^2 ds$$

$$= e^{-x} \left[e^s (s^2 - 2s + 2) \right]_0^x$$

(6)

$$= e^{-x} \{ e^x (x^2 - 2x + 2) - 1 \times 2 \} = \boxed{(x^2 - 2x + 2) - 2e^{-x}}$$

$$(iii) \quad \textcircled{1} \Leftrightarrow W'(x) = f(x) - aW(x) \cdots \cdots \textcircled{2}$$

x で微分すると

$$W''(x) = f'(x) - aW'(x)$$

②と $f'(x) = g(x)$ を代入すると

$$W''(x) = g(x) - a(f(x) - aW(x))$$

$$\boxed{= a^2W(x) - af(x) + g(x)}$$

(7)

4

$$m \frac{d^2x_1}{dt^2} = -k(x_2 - x_1) + kl \cdots \cdots \textcircled{1}$$

$$m \frac{d^2x_2}{dt^2} = k(x_2 - x_1) - kl + F \cdots \cdots \textcircled{2}$$

(i) $x_2 - x_1 = u$ とおく

$$\frac{du}{dt} = \frac{dx_2}{dt} - \frac{dx_1}{dt}, \quad \frac{d^2u}{dt^2} = \frac{d^2x_2}{dt^2} - \frac{d^2x_1}{dt^2} \cdots \cdots \textcircled{3}$$

② - ①より

$$m \left(\frac{d^2x_2}{dt^2} - \frac{d^2x_1}{dt^2} \right) = 2k(x_2 - x_1) - 2kl + F$$

$$\textcircled{3} \Leftrightarrow m \frac{d^2u}{dt^2} = \boxed{2ku - 2kl + F} \quad (8)$$

(ii) $x_2 + x_1 = v$ とおく

$$\frac{dv}{dt} = \frac{dx_2}{dt} + \frac{dx_1}{dt}, \quad \frac{d^2v}{dt^2} = \frac{d^2x_2}{dt^2} + \frac{d^2x_1}{dt^2} \cdots \cdots \textcircled{4}$$

② + ①より

$$m \left(\frac{d^2x_2}{dt^2} + \frac{d^2x_1}{dt^2} \right) = F$$

$$\textcircled{4} \Leftrightarrow m \frac{d^2v}{dt^2} = \boxed{F} \quad (9)$$

$$(iii) \quad \boxed{W''(x) = a^2W(x) - af(x) + f'(x) \cdots \cdots \textcircled{5}}$$

$$\Rightarrow W(x) = \int_0^x e^{-a(x-s)} f(s) ds$$

道具

u について

$$m \frac{d^2 u}{dt^2} = 2ku - 2kl + F = 2ku - 2kl + c$$

$$u''(t) = \frac{2k}{m}u + \frac{c - 2kl}{m}$$

$$a^2 = \frac{2k}{m} \text{ とおくと } a = \pm \sqrt{\frac{2k}{m}}$$

$$-af(x) + f'(x) = \frac{c - 2kl}{m}$$

x で積分 $\left\{ \begin{array}{l} -ae^{-ax}f(x) + e^{-ax}f'(x) = \frac{c - 2kl}{m}e^{-ax} \\ e^{-ax}f(x) = -\frac{c - 2kl}{ma}e^{-ax} + E \end{array} \right.$

(E :積分定数)

$$f(x) = -\frac{c - 2kl}{ma} + Ee^{ax}$$

$$\begin{aligned} u &= \int_0^t e^{-a(t-s)} f(s) ds \\ &= \int_0^t e^{-a(t-s)} \times \left\{ -\frac{c - 2kl}{ma} + Ee^{as} \right\} ds \\ &= e^{-at} \int_0^t e^{as} \left(-\frac{c - 2kl}{ma} + Ee^{as} \right) ds \\ &= e^{-at} \int_0^t \left(-\frac{c - 2kl}{ma} e^{as} + Ee^{2as} \right) ds \\ &= e^{-at} \left[-\frac{c - 2kl}{ma^2} e^{as} + \frac{E}{2a} e^{2as} \right]_0^t \\ &= e^{-at} \left\{ \left(-\frac{c - 2kl}{ma^2} e^{at} + \frac{E}{2a} e^{2at} \right) - \left(-\frac{c - 2kl}{ma^2} + \frac{E}{2a} \right) \right\} \\ &= -\frac{c - 2kl}{ma^2} + \frac{c - 2kl}{ma^2} e^{-at} + \frac{E}{2a} (e^{at} - e^{-at}) \end{aligned}$$

u が発散しないので, $E = 0$, $a > 0$ より $a = \sqrt{\frac{2k}{m}}$

$$u(t) = -\frac{c - 2kl}{m \times \frac{2k}{m}} + \frac{c - 2kl}{m \times \frac{2k}{m}} e^{-\sqrt{\frac{2k}{m}} t}$$

$$= \frac{c-2kl}{2k} + \frac{c-2kl}{2k} e^{-\sqrt{\frac{2k}{m}}t}$$

(10)

$v(x)$ について

$$m \frac{d^2v}{dt^2} = F = c \Leftrightarrow V''(x) = \frac{c}{m}$$

⑤において $a=0$

$$-af(x) + f'(x) = \frac{c}{m}$$

$$f'(x) = \frac{c}{m} \quad f(x) = \frac{c}{m}x + D \quad (D: \text{積分定数})$$

$$\begin{aligned} v(t) &= \int_0^t e^{-0(t-s)} f(s) ds = \int_0^t f(s) ds \\ &= \int_0^t \left(\frac{c}{m}s + D \right) ds \\ &= \left[\frac{c}{2m}s^2 + Ds \right]_0^t \\ &= \frac{c}{2m}t^2 + Dt \quad (D: \text{積分定数}) \end{aligned}$$

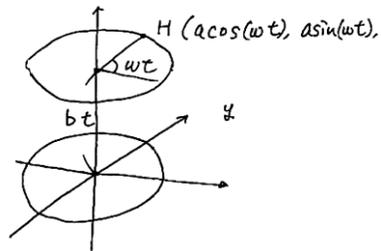
(11)

5

$$x = x(t) = a \cos(\omega t)$$

$$y = y(t) = a \sin(\omega t)$$

$$z = z(t) = bt$$



$$\begin{aligned} \text{(i)} \quad \text{OH}^2 &= a^2 \cos^2(\omega t) + a^2 \sin^2(\omega t) + b^2 t^2 \\ &= a^2 + b^2 t^2 \end{aligned}$$

(12)

$$\therefore \text{OH} = \sqrt{a^2 + b^2 t^2}$$

$$\text{(ii)} \quad \text{OH} = d$$

$$\sqrt{a^2 + b^2 t_d^2} = d$$

$$a^2 + b^2 t_d^2 = d^2$$

$$t^2 d = \frac{d^2 - a^2}{b^2}$$

$$td \geq 0 \text{ より } td = \frac{\sqrt{d^2 - a^2}}{b} \quad (13)$$

$$\begin{aligned}
 \text{(iii)} \quad l &= \int_0^{td} \sqrt{x'(t)^2 + y'(x)^2 + z'(t)^2} dt & x'(t) &= -awsin wt \\
 & & t'(t) &= awcoswt \\
 & & z'(t) &= b \\
 &= \int_0^{td} \sqrt{a^2 w^2 \sin^2 wt + a^2 w^2 \cos^2 wt + b^2} dt \\
 &= \int_0^{td} \sqrt{a^2 w^2 + b^2} dt \\
 &= \sqrt{a^2 w^2 + b^2} [t]_0^{td} \\
 &= \sqrt{a^2 w^2 + b^2} td \\
 &= \sqrt{a^2 w^2 + b^2} \times \frac{\sqrt{d^2 - a^2}}{b} \\
 &= \frac{\sqrt{(a^2 w^2 + b^2)(d^2 - a^2)}}{b} \\
 & \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad |\overrightarrow{PQ}|^2 &= \{a \cos(w(t+h)) - a \cos(wt)\}^2 + \{a \sin(w(t+h)) - a \sin(wt)\}^2 \\
 &= a^2 \{(\cos(w(t+h)) - \cos(wt))^2 + (\sin(w(t+h)) - \sin(wt))^2\} \\
 |z(t+h) - z(t)| &= |b(t+h) - bt| = |bh| \\
 \lim_{h \rightarrow 0} \frac{|z(t+h) - z(t)|^2}{|\overrightarrow{PQ}|^2} & \\
 &= \lim_{h \rightarrow 0} \frac{b^2 h^2}{a^2 \{(\cos w(t+h) - \cos wt)^2 + (\sin w(t+h) - \sin wt)^2\}} \\
 &= \lim_{h \rightarrow 0} \frac{b^2}{\left\{ \left(\frac{\cos w(t+h) - \cos wt}{h} \right)^2 + \left(\frac{\sin w(t+h) - \sin wt}{h} \right)^2 \right\}} \\
 &= \frac{b^2}{a^2 \{(-w \sin wt)^2 + (w \cos wt)^2\}} \\
 &= \frac{b^2}{a^2 w^2} \\
 \therefore \lim_{h \rightarrow 0} \frac{|z(t+h) - z(t)|}{|\overrightarrow{PQ}|} &= \sqrt{\frac{b^2}{a^2 w^2}} = \frac{b}{aw} \\
 & \quad (15)
 \end{aligned}$$